PC02 — Kinematics & Laws of Motion

📖 PC02 · CDS General Science — Physics

🎯 CDS Level : High Priority

This is the highest-yield chapter in CDS Physics. Kinematics describes *how* objects move; Newton's laws explain *why*. Expect 2–3 questions from here in every CDS paper — covering equations of motion, projectile motion, Newton's three laws, impulse-momentum, and friction.

📌 **CDS regularly tests:** Equations of motion (especially $v^2 = u^2 + 2as$); projectile range and height; complementary angles giving same range; gun-bullet recoil (conservation of momentum); Newton's 2nd law numericals; friction types; circular motion centripetal force; apparent weight in a lift.

Topics at a Glance

① Scalars & Vectors
Distance vs displacement; speed vs velocity

② Equations of Motion
$v=u+at$; $s=ut+\frac{1}{2}at^2$; $v^2=u^2+2as$

③ Motion Graphs
s–t and v–t graphs; slope; area

④ Projectile Motion
Range, height, time of flight; 45° max range

⑤ Newton's Laws & Friction
Inertia; F=ma; action-reaction; μ types

⑥ Momentum & Circular Motion
Conservation of momentum; centripetal force

1. Scalars, Vectors & Equations of Motion

1.1 Types of Motion & Equations Under Uniform Acceleration

Scalars vs Vectors

- **Scalar:** magnitude only — distance, speed, mass, time, energy, temperature
- **Vector:** magnitude + direction — displacement, velocity, force, acceleration, momentum
- Vector addition: triangle law or parallelogram law
- Resultant of two equal vectors at angle θ: $R = 2A\cos(\theta/2)$

Distance vs Displacement

- Distance = total path length (scalar; always positive)
- Displacement = shortest path from start to end (vector; can be negative)
- Speed = distance / time (scalar)
- Velocity = displacement / time (vector)
- Average speed ≥ average velocity (always)

THREE EQUATIONS OF MOTION (UNIFORM ACCELERATION)

$v = u + at$ ← find velocity after time t

$s = ut + \frac{1}{2}at^2$ ← find displacement in time t

$v^2 = u^2 + 2as$ ← find velocity given displacement (no t needed)

$s_n = u + \frac{1}{2}a(2n - 1)$ ← displacement in the nth second

Free fall (downward): use $a = +g = 9.8\ m/s^2 \approx 10\ m/s^2$

Throw upward: use $a = -g$ (decelerating); at max height $v = 0$

Time to go up = time to come down (symmetric motion under gravity)

Use $v^2 = u^2 + 2as$ whenever time is not given and not required — saves significant calculation time in CDS.

2. Motion Graphs

2.1 Reading s-t and v-t Graphs

CDS often gives a graph and asks for velocity, acceleration, or displacement

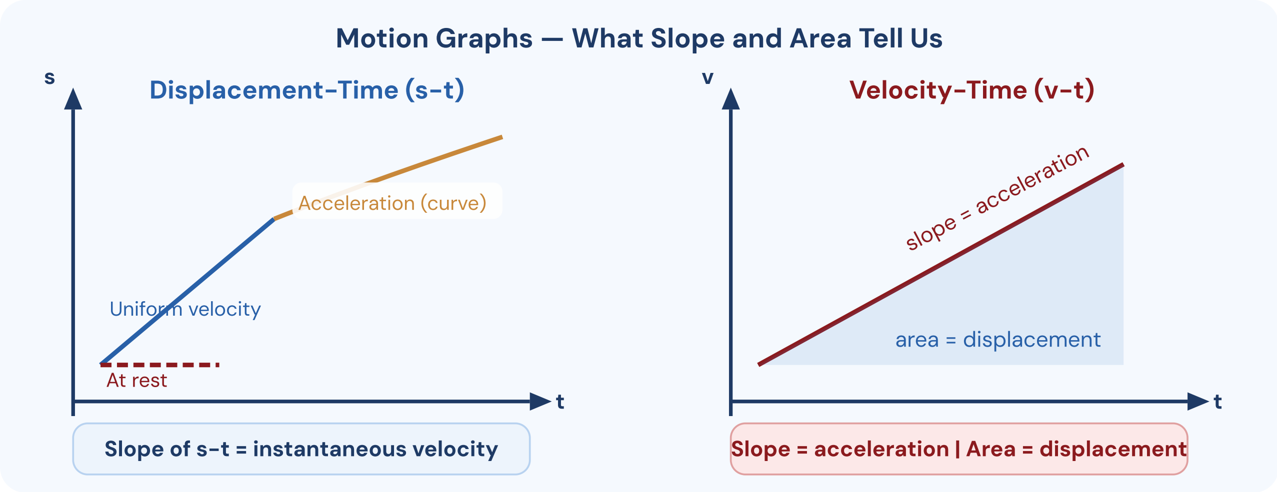

Fig. 1 — Motion graph rules: slope of s-t gives velocity; slope of v-t gives acceleration; area under v-t gives displacement. These are direct CDS graph-reading questions.

3. Projectile Motion

3.1 Two-Dimensional Motion Under Gravity

PROJECTILE FORMULAE

Horizontal component: $u_x = u \cos\theta$ (constant — no horizontal force)

Vertical component: $u_Y = u \sin\theta$ (reduces under gravity)

Time of flight: $T = 2u \sin\theta / g$

Maximum height: $H = u^2 \sin^2\theta / 2g$

Horizontal range: $R = u^2 \sin 2\theta / g$

Maximum range at $\theta = 45°$: $R_{max} = u^2/g$

At 45°: $H = R/4$ (height is exactly one-quarter of range)

Complementary angles give same range:

30° & 60° → same R; 20° & 70° → same R; 15° & 75° → same R

At any time t: $x = u \cos\theta \cdot t$; $y = u \sin\theta \cdot t - \frac{1}{2}gt^2$

Key insight: Horizontal and vertical motions are completely independent. A ball thrown horizontally and a ball dropped vertically from the same height both hit the ground at exactly the same time — horizontal speed has zero effect on the time of fall.

4. Newton's Laws of Motion & Friction

4.1 Three Laws, Impulse, Momentum & Friction Types

① First Law — Inertia

- Body at rest stays at rest; body moving stays moving — until net external force acts
- Inertia ∝ mass
- Examples: passenger jerks forward when bus brakes; coin on card; seat belts

② Second Law — F = ma

- **F = ma** (net force = mass × acceleration)
- $F = dp/dt$ (rate of change of momentum)
- Impulse = $F \times t = \Delta p = m(v-u)$
- Apparent weight in lift: $N = m(g \pm a)$

③ Third Law — Action-Reaction

- Every action has equal and opposite reaction — on different bodies
- Gun recoil; rocket propulsion; swimming; boat-oar
- They never cancel (act on different objects)

IMPULSE, MOMENTUM & FRICTION

Momentum: $p = mv$ $[MLT^{-1}]$

Impulse: $J = F \cdot t = \Delta p$ $[MLT^{-1}]$ (area under F–t graph)

Conservation of momentum (when net external force = 0):

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ (for all collisions)

Gun-bullet example:

Initial momentum = 0; $m_b \cdot v_b + m_g \cdot v_g = 0$ → gun recoils

Friction:

$f\_max = \mu_s N$ (limiting static friction; μ_s = coefficient of static friction)

Kinetic friction = $\mu_k N$ (while sliding)

Order of magnitude: $\mu_s > \mu_k > \mu\_rolling$

Friction is independent of surface area of contact

⚠ **CDS Traps in Kinematics & Laws:** (1) Complementary angles give same range — 30° and 60°, NOT 30° and 45°. (2) At max height of projectile: v_vertical = 0, but v_horizontal = u cosθ (not zero). (3) Friction is independent of area — only depends on normal force and surfaces. (4) Inertia is measured by mass, not weight. (5) In free fall, apparent weight = 0 (weightlessness), actual weight unchanged.

5. Circular Motion

5.1 Uniform Circular Motion — Speed Constant, Direction Always Changing

CIRCULAR MOTION FORMULAE

Angular velocity: $\omega = 2\pi/T = 2\pi f$ (rad/s)

Linear velocity: $v = \omega r$

Centripetal acc: $a_c = v^2/r = \omega^2 r$ (directed toward centre)

Centripetal force: $F_c = mv^2/r$ (provided by: gravity/tension/friction)

Work done by centripetal force = 0 (F always ⊥ v)

Centrifugal force is a *pseudo-force* felt in a rotating (non-inertial) frame — it does not exist in an inertial frame. A passenger in a turning car feels pushed outward — that sensation is inertia, not a real outward force.

Q1. A gun of mass 3 kg fires a bullet of mass 30 g with velocity 300 m/s. The recoil velocity of the gun is:

(a) 1 m/s

(b) 3 m/s

(c) 0.3 m/s

(d) 30 m/s

Answer: (b) 3 m/s
By conservation of momentum (initial momentum = 0):
$0 = m_b \cdot v_b + m_g \cdot v_g \rightarrow 0 = 0.030 \times 300 + 3 \times v_g \rightarrow v_g = -9/3 =$ **-3 m/s**
(negative = opposite to bullet). Gun recoils at 3 m/s.

Q2. A ball is thrown at 45° and travels horizontal range R. The maximum height reached is:

(a) R

(b) R/2

(c) R/4

(d) 2R

Answer: (c) R/4
At $\theta = 45°$: $H = u^2\sin^2 45°/2g = u^2/4g$ and $R = u^2\sin 90°/g = u^2/g$. Therefore H = R/4. This specific ratio at 45° is directly and repeatedly tested in CDS — commit it to memory.

Q3. A body starts from rest and acquires velocity v in time t. The distance covered is:

(a) vt

(b) vt/2

(c) 2vt

(d) $vt^2/2$

Answer: (b) vt/2
From rest: u = 0; final velocity = v; time = t. From v = u + at → a = v/t. Distance s = ut +

$\frac{1}{2}at^2 = 0 + \frac{1}{2}(v/t)t^2 = vt/2$. Alternatively: $s = (u+v)/2 \times t = (0+v)/2 \times t =$ **$vt/2$**. The average velocity method is fastest here.

Q4. The horizontal range of a projectile is the same for angles of projection:

(a) 20° and 60°

(b) 30° and 60°

(c) 30° and 45°

(d) 45° and 60°

Answer: (b) 30° and 60°
Range $R = u^2\sin 2\theta/g$ is the same for complementary angles (θ and 90°−θ). 30° + 60° = 90° → complementary → same range. Similarly 20° & 70°, 15° & 75°. Neither 20°&60°, nor 30°&45°, nor 45°&60° are complementary pairs.

📚 Formula Sheet — PC02

📊 Equations of Motion

- ∴ $v = u + at$
- ∴ $s = ut + \frac{1}{2}at^2$
- ∴ $v^2 = u^2 + 2as$
- ∴ $s_n = u + \frac{1}{2}a(2n-1)$
- ∴ Avg velocity = $(u+v)/2$

🎯 Projectile

- ∴ $T = 2u\sin\theta/g$
- ∴ $H = u^2\sin^2\theta/2g$
- ∴ $R = u^2\sin 2\theta/g$; max at 45°
- ∴ Complementary angles → same R
- ∴ At 45°: $H = R/4$

👥 Newton's Laws

- ∴ $F = ma$; $F = dp/dt$
- ∴ $p = mv$; $J = F\Delta t = \Delta p$
- ∴ $\mu_s > \mu_k > \mu_{rolling}$
- ∴ $f = \mu N$ (independent of area)

🔄 Circular Motion

- ∴ $v = \omega r$; $\omega = 2\pi f = 2\pi/T$
- ∴ $a_c = v^2/r$ (centripetal, inward)
- ∴ $F_c = mv^2/r$
- ∴ Work done by centripetal force = 0

- Lift: N = m(g±a)
- Centrifugal = pseudo-force only

Quick Revision — PC02

Key Traps

- Max height at 45°: v_horizontal ≠ 0
- Dropped + thrown horizontal: same fall time
- Inertia ∝ mass (not weight)
- Friction: area doesn't matter
- H = R/4 always at 45°

Projectile Facts

- Max range at 45°
- Complementary angles: same range
- 30° & 60°; 20° & 70°; 15° & 75°
- Horizontal: constant velocity
- Vertical: uniform acceleration g

Graph Rules

- s-t slope = velocity
- v-t slope = acceleration
- v-t area = displacement
- Horizontal v-t = constant velocity
- Curved s-t = non-uniform motion

Practice Exercise

E-01

A body is thrown vertically upward with 20 m/s. Maximum height ($g=10\ m/s^2$):

(a) 10 m
(b) 20 m
(c) 40 m
(d) 5 m

E-02

A 5 kg body accelerates at $4\ m/s^2$. The net force acting on it is:

(a) 1.25 N
(b) 20 N

(c) 9 N

(d) 0.8 N

E-03

A ball is projected at 60° with velocity 20 m/s. Time of flight ($g=10\ m/s^2$):

(a) 2 s

(b) $2\sqrt{3}$ s

(c) 4 s

(d) 1 s

Answers: E-01: (b) 20 m [$v^2=u^2-2gH$; $0=400-20H$; $H=20m$] | E-02: (b) 20 N [$F=ma=5\times4=20$ N] | E-03: (b) $2\sqrt{3}$ s [$T=2u\sin\theta/g=2\times20\times\sin60°/10=4\times(\sqrt{3}/2)=2\sqrt{3}$ s]

Mock Tests | Subject Quizzes | Telegram