Complex Numbers

📘 Algebra · Chapter MN02 🎯 NDA Level : High Priority

Complex numbers extend the real number system to solve equations like $x^2 + 1 = 0$ — which have no real solution. For NDA Mathematics, this chapter is consistently tested every year. Questions are concept-heavy but calculation-light, making it ideal for quick high-yield preparation. The three pillars are: **powers of iota**, **modulus/argument/conjugate**, and **cube roots of unity**.

> 📌 **What to expect in NDA (based on 2022–2024 papers):**
> **(1)** Finding i^n for large n using the cycle of 4; **(2)** Modulus $|z| = \sqrt{(a^2+b^2)}$; **(3)** Conjugate and its properties; **(4)** Simplifying expressions using $1+\omega+\omega^2 = 0$ and $\omega^3 = 1$; **(5)** Value of $(1+\omega)$^n or $(1+\omega^2)$^n; **(6)** Real and imaginary parts of a given complex expression; **(7)** Argument of a complex number; **(8)** Square root of a complex number (less frequent but tested).

Topics at a Glance

① Iota & Powers
i, i^2, i^3, i^4 — cycle of 4, finding i^n

② Cartesian Form
z = a + ib, real part, imaginary part

③ Modulus
$|z| = \sqrt{(a^2+b^2)}$, properties

④ Argument & Conjugate
$\theta = \arg(z)$, $\bar{z} = a - ib$

⑤ Square Root of z
$\sqrt{(a+ib)}$ — method via x+iy

⑥ Cube Roots of Unity
$\omega, \omega^2, 1+\omega+\omega^2=0, \omega^3=1$

1. Iota (i) — The Imaginary Unit & Its Powers

1.1

Definition & the Cycle of Four

Highest frequency topic — asked almost every NDA paper

The imaginary unit **i** (iota) is defined as **$i = \sqrt{-1}$**, so $i^2 = -1$. This single definition generates an endlessly repeating 4-step cycle when you raise i to successive positive integer powers.

i^1	i^2	i^3	i^4
$= i$	$= -1$	$= -i$	$= 1$

After $i^4 = 1$, the cycle restarts: $i^5 = i$, $i^6 = -1$, and so on. The pattern has a **period of 4**.

FINDING I^N FOR ANY POSITIVE INTEGER N

Step 1: Divide n by 4 and find the remainder r = n mod 4

Step 2: i^n = i^r

r = 0 → i^n = 1 (e.g., $i^{40} = 1$)
r = 1 → i^n = i (e.g., $i^{41} = i$)
r = 2 → i^n = −1 (e.g., $i^{42} = -1$)
r = 3 → i^n = −i (e.g., $i^{43} = -i$)

Special: $i^0 = 1$. For negative powers: $i^{-1} = -i$, $i^{-2} = -1$, $i^{-3} = i$, $i^{-4} = 1$.

WORKED EXAMPLE — LARGE POWER OF I

Find i^{997}.

$997 \div 4 = 249$ remainder **1**. So $i^{997} = i^1 =$ **i**.

Find i^{-3}.

$i^{(-3)} = 1/i^3 = 1/(-i) = -(1/i) = -(i/i^2) = -(i/-1) = i$. Or: $i^{(-3)} = i^{(4-3)} = i^1 =$ **i**. (Add 4 until positive.)

⚠ Common Traps with Iota:

- **$i^{4k} = 1$** for any integer k (including k=0). Students sometimes write $i^4 = i$ — wrong.
- $\sqrt{-a} = i\sqrt{a}$ only if $a > 0$. **$\sqrt{-4} = 2i$, NOT $-2i$ or $\sqrt{4}\cdot\sqrt{-1}$.**
- $i^2 = -1$ (not +1). This is the most basic and most often misapplied fact.

📌 **Sum of consecutive powers:** i^n + i^(n+1) + i^(n+2) + i^(n+3) = 0 always (sum of one full cycle = 0). This is sometimes asked directly.

📋 TOPIC-WISE PYQ

Powers of Iota — NDA-Pattern Questions

Q1. The value of $i^{57} + i^{58} + i^{59} + i^{60}$ is:

(a) 0 (b) i (c) -1 (d) 1

Answer: (a) 0
Sum of any 4 consecutive powers of i is always 0. Here remainders of 57,58,59,60 mod 4 are 1,2,3,0 → i + (−1) + (−i) + 1 = **0**.

Q2. What is the value of i^(4n+3) where n is any positive integer?

(a) i (b) −i (c) −1 (d) 1

Answer: (b) −i
4n+3 divided by 4 always gives remainder 3. So i^(4n+3) = i^3 = **−i**.

Q3. The value of i^(−999) is:

(a) i (b) −1 (c) −i (d) 1

Answer: (a) i
i^(−999) = i^(−999 + 1000) = i^1 = **i**. (Add the smallest multiple of 4 that makes the power non-negative: 999 = 4×249+3, so −999 ≡ 1 mod 4 → i^1 = i.)

🔥 TRICKY QUESTIONS

Iota — Classic NDA Traps

🧩 **T1. Find the value of $i^1 + i^2 + i^3 + \ldots + i^{100}$.**

Solution: 0.
100 is divisible by 4, so there are exactly 25 complete cycles. Each cycle ($i^1+i^2+i^3+i^4$) =

i+(−i)+(−i)+1 = 0. Total = 25 × 0 = **0**.

Trap: Trying to add term by term instead of grouping in cycles of 4.

T2. Is $\sqrt{-1} \times \sqrt{-1} = \sqrt{-1 \times -1} = \sqrt{1} = 1$?

Solution: NO. The answer is −1, not 1.

$\sqrt{-1} \times \sqrt{-1} = i \times i = i^2 = -1$. The rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ only holds when a, b ≥ 0. It CANNOT be applied to negative numbers.

This is a conceptual trap that appears in statement-based NDA questions.

2. Cartesian Form, Modulus, Argument & Conjugate

2.1

Cartesian Form — a + ib

The standard representation — everything else builds on this

Every complex number z is written as **z = a + ib**, where a and b are real numbers.

Real & Imaginary Parts

- **Re(z) = a** — the real part
- **Im(z) = b** — the imaginary part (b is real!)
- z is *purely real* if b = 0 (e.g., z = 5)
- z is *purely imaginary* if a = 0 (e.g., z = 3i)
- z = 0 iff a = 0 AND b = 0 simultaneously

Equality of Complex Numbers

- If a + ib = c + id, then a = c AND b = d
- Two complex numbers are equal iff both real parts and imaginary parts match
- No concept of "greater than" or "less than" between complex numbers
- Only equality/inequality of moduli can be compared

BASIC ARITHMETIC OF COMPLEX NUMBERS

Addition: $(a+ib) + (c+id) = (a+c) + i(b+d)$

Subtraction: $(a+ib) - (c+id) = (a-c) + i(b-d)$

Multiplication: $(a+ib)(c+id) = (ac-bd) + i(ad+bc)$

Division: $(a+ib)/(c+id) = (a+ib)(c-id) / (c^2+d^2)$

Division method: Multiply numerator and denominator by the conjugate of the denominator (c−id) to make the denominator real.

WORKED EXAMPLE — DIVISION

Simplify (3+4i)/(1+2i).

Multiply top and bottom by conjugate of denominator (1−2i):

$= (3+4i)(1-2i) / (1+2i)(1-2i)$

$= (3-6i+4i-8i^2) / (1+4)$

$= (3-2i+8) / 5$ [since $i^2=-1$, so $-8i^2=+8$]

$= \mathbf{(11-2i)/5 = 11/5 - (2/5)i}$

2.2

Modulus — Distance from Origin

|z| appears in almost every complex number MCQ

MODULUS DEFINITION & PROPERTIES

$|z| = |a + ib| = \sqrt{a^2 + b^2}$ (always ≥ 0)

$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

$|z_1 / z_2| = |z_1| / |z_2|$

$|z_1 + z_2| \leq |z_1| + |z_2|$ (Triangle Inequality)

$|z_1 - z_2| \geq ||z_1| - |z_2||$ (Reverse Triangle Inequality)

$|z|^2 = z \cdot \bar{z} = a^2 + b^2$

$|z| = |\bar{z}|$ (modulus equals modulus of conjugate)

For NDA: The triangle inequality is frequently tested in statement-based questions. $|z|^2 = z\bar{z}$ is very useful in division problems.

Examples of Modulus

- $|3 + 4i| = \sqrt{9+16} = \sqrt{25} = 5$
- $|1 + i| = \sqrt{1+1} = \sqrt{2}$
- $|-5| = 5$ (purely real: b=0)
- $|7i| = 7$ (purely imaginary: a=0)
- $|0| = 0$

On the Argand Plane

- |z| = distance from origin O to point P(a,b)
- $|z_1 - z_2|$ = distance between points P_1 and P_2
- All z with |z| = r lie on a circle radius r centred at O

- $|z| = 1 \to$ unit circle

2.3

Argument — Angle in the Argand Plane

θ = arg(z) — principal value lies in (−π, π]

The **argument** of $z = a + ib$ is the angle θ that the line OP makes with the positive real axis, measured anticlockwise. The **principal argument** is the unique value in (−π, π].

Quadrant / Case	Condition	Argument θ (Principal)
1st Quadrant	$a > 0, b > 0$	$\theta = \arctan(b/a)$ (positive, acute)
2nd Quadrant	$a < 0, b > 0$	$\theta = \pi - \arctan(b/|a|)$
3rd Quadrant	$a < 0, b < 0$	$\theta = -\pi + \arctan(|b|/|a|)$
4th Quadrant	$a > 0, b < 0$	$\theta = -\arctan(|b|/a)$
Positive real axis	$b = 0, a > 0$	$\theta = 0$
Negative real axis	$b = 0, a < 0$	$\theta = \pi$
Positive imaginary	$a = 0, b > 0$	$\theta = \pi/2$
Negative imaginary	$a = 0, b < 0$	$\theta = -\pi/2$

📌 **Quick Memory Aid:** $\arg(1) = 0$, $\arg(i) = \pi/2$, $\arg(-1) = \pi$, $\arg(-i) = -\pi/2$. And $\arg(z_1 \cdot z_2) = \arg(z_1) + \arg(z_2)$.

2.4

Conjugate — Reflection Across Real Axis

Properties tested in both standalone MCQs and within larger problems

⚡ **CONJUGATE DEFINITION & PROPERTIES**

If $z = a + ib$, then $\bar{z}$ (conjugate) $= a - ib$

$z + \bar{z} = 2a = 2 \cdot \text{Re}(z)$ (purely real)

$z - \bar{z} = 2ib = 2i \cdot \text{Im}(z)$ (purely imaginary)

$z \cdot \bar{z} = a^2 + b^2 = |z|^2$ (always real and positive)

$\overline{(\bar{z})} = z$ (double conjugate returns original)

$\bar{z}_1 + \bar{z}_2 = \overline{(z_1+z_2)}$ (conjugate of sum = sum of conjugates)

$\bar{z}_1 \cdot \bar{z}_2 = \overline{(z_1 \cdot z_2)}$ (conjugate of product = product of conjugates)

z is purely real $\Leftrightarrow z = \bar{z}$. z is purely imaginary $\Leftrightarrow z = -\bar{z}$. These two conditions are directly tested.

Argand Plane — Geometric View

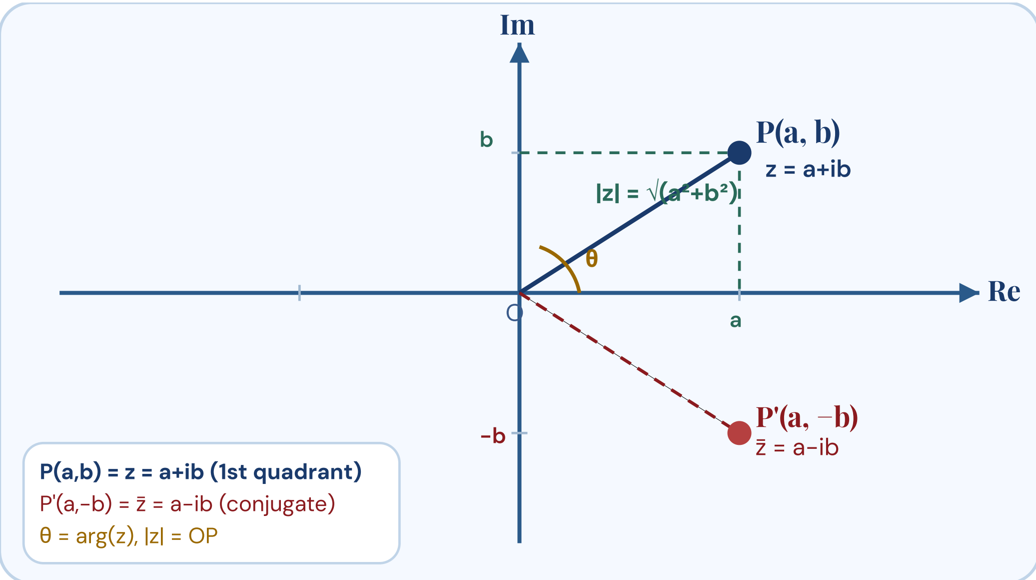

Fig 1: Argand Plane — z and its conjugate z̄ as mirror images across the real axis; |z| is the distance OP; θ is the argument.

TOPIC-WISE PYQ **Modulus, Argument & Conjugate — NDA-Pattern Questions**

Q4. If $z = 3 - 4i$, then $|z|$ is:

(a) 1 (b) 5 (c) 7 (d) 25

Answer: (b) 5

$|z| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = \mathbf{5}$. Note: modulus uses squares, so the negative sign on Im(z) doesn't affect it.

Q5. The conjugate of $(2+3i)/(1-i)$ is:

(a) $(-1+5i)/2$ (b) $(-1-5i)/2$ (c) $(1-5i)/2$ (d) $(1+5i)/2$

Answer: (b) (−1−5i)/2
First simplify: multiply by conjugate of denominator (1+i)/(1+i).
= (2+3i)(1+i) / (1−i)(1+i) = (2+2i+3i+3i²) / (1+1) = (2+5i−3)/2 = (−1+5i)/2.
Conjugate = **(−1−5i)/2**.

Q6. The argument of the complex number $-1 + i\sqrt{3}$ is:

(a) π/3 (b) 2π/3 (c) π/6 (d) −2π/3

Answer: (b) 2π/3
Here a = −1, b = √3 → 2nd quadrant. Basic angle = arctan(√3/1) = π/3. Argument = π − π/3 = **2π/3**.

Q7. If $z \cdot \bar{z} = 25$ and $z = 3 + bi$, find b.

(a) 2 (b) 3 (c) 4 (d) 5

Answer: (c) 4
$z \cdot \bar{z} = |z|^2 = 3^2 + b^2 = 9 + b^2 = 25 \rightarrow b^2 = 16 \rightarrow$ **b = 4**.

🔥 TRICKY QUESTIONS

Modulus & Conjugate — Deceptive MCQs

T3. If $|z_1| = |z_2|$, does it follow that $z_1 = z_2$?

NO. Equal moduli only means both points are the same distance from the origin (lie on the same circle). For example, |3+4i| = |3−4i| = 5, but 3+4i ≠ 3−4i.
Trap: Students assume equal modulus means equal complex numbers. This is a statement-verification question in NDA.

T4. z is purely imaginary. Then $z + \bar{z}$ = ?

0. $z + \bar{z} = 2 \cdot \mathrm{Re}(z) = 2a$. If z is purely imaginary, a = 0. So $z + \bar{z} = 0$.
Corollary: z is purely real ⇔ $z = \bar{z}$. z is purely imaginary ⇔ $z = -\bar{z}$. Both conditions appear as direct MCQs.

T5. $|z + \bar{z}|$ = ? vs $|z| + |\bar{z}|$

$z + \bar{z} = 2a$ (purely real), so $|z + \bar{z}| = 2|a|$.
$|z| + |\bar{z}| = 2|z|$ (since $|z| = |\bar{z}| = \sqrt{a^2+b^2}$).
These are equal only when b = 0 (z is real). In general, $2|a| \le 2\sqrt{a^2+b^2}$.
This tests Triangle Inequality + conjugate properties together.

3. Square Root of a Complex Number

3.1

Method: Let $\sqrt{a + ib} = x + iy$

Set up two equations — solve systematically

To find $\sqrt{a + ib}$, assume it equals x + iy where x, y are real. Squaring both sides and comparing real and imaginary parts gives a solvable system.

SQUARE ROOT METHOD — STEP BY STEP

Let $\sqrt{a + ib} = x + iy$

Square both sides: $a + ib = (x^2 - y^2) + i(2xy)$

Comparing parts:

$x^2 - y^2 = a$ ...(1)

$2xy = b$ ...(2)

Also use: $x^2 + y^2 = \sqrt{a^2 + b^2} = |z|$ ...(3)

From (1) and (3):

$x^2 = (|z| + a)/2 \rightarrow x = \pm\sqrt{[(|z| + a)/2]}$

$y^2 = (|z| - a)/2 \rightarrow y = \pm\sqrt{[(|z| - a)/2]}$

Sign of y is determined by equation (2): $2xy = b$

If $b > 0$: x and y have same sign

If $b < 0$: x and y have opposite signs

There are always two square roots: +(x + iy) and −(x + iy). Both are valid.

WORKED EXAMPLE — SQUARE ROOT

Find $\sqrt{3 + 4i}$.

Here $a = 3$, $b = 4$. $|z| = \sqrt{9+16} = 5$.

$x^2 = (5+3)/2 = 4 \rightarrow x = \pm 2$.

$y^2 = (5-3)/2 = 1 \rightarrow y = \pm 1$.

Since $b = 4 > 0$, x and y same sign $\rightarrow x = 2, y = 1$ or $x = -2, y = -1$.

$\sqrt{3+4i} = 2 + i$ or $-(2+i) = -2-i$.

⚠ **Exam Note:** The square root of a complex number always gives *two* values (just like square roots of real numbers). If the question asks for "the square root", both $\pm(x+iy)$ are answers. NDA occasionally gives this as a 4-option MCQ — choose the correct pair.

TOPIC-WISE PYQ **Square Root of Complex Number — NDA-Pattern Questions**

Q8. The square roots of $-7 + 24i$ are:

(a) $\pm(3 + 4i)$ (b) $\pm(4 + 3i)$ (c) $\pm(3 - 4i)$ (d) $\pm(4 - 3i)$

Answer: (a) $\pm(3 + 4i)$

$|z| = \sqrt{49+576} = \sqrt{625} = 25$. $x^2 = (25-7)/2 = 9 \rightarrow x = \pm 3$. $y^2 = (25+7)/2 = 16 \rightarrow y = \pm 4$. Since $b = 24 > 0$, same signs. $\sqrt{-7+24i} = $ **$\pm(3+4i)$**. Verify: $(3+4i)^2 = 9+24i+16i^2 = 9+24i-16 = -7+24i$ ✓.

4. Cube Roots of Unity — ω and Its Properties

4.1 Finding the Cube Roots of Unity

One real root, two complex roots — know all three by heart

The cube roots of unity are the three solutions of $\mathbf{x^3 = 1}$, equivalently $\mathbf{x^3 - 1 = 0}$.

DERIVATION — CUBE ROOTS OF 1

$x^3 - 1 = 0$

$(x - 1)(x^2 + x + 1) = 0$

Root 1: $x = 1$ (the real cube root)

Roots 2 & 3 from $x^2 + x + 1 = 0$ (quadratic formula):

$x = [-1 \pm \sqrt{(1-4)}] / 2 = [-1 \pm \sqrt{(-3)}] / 2 = [-1 \pm i\sqrt{3}] / 2$

$\omega = (-1 + i\sqrt{3}) / 2$ $|\omega| = 1$, $\arg(\omega) = 2\pi/3$

$\omega^2 = (-1 - i\sqrt{3}) / 2$ $|\omega^2| = 1$, $\arg(\omega^2) = 4\pi/3 = -2\pi/3$

Key: ω and ω^2 are complex conjugates of each other: $\bar{\omega} = \omega^2$.

📌 **Memory Aid:** ω = "half of $(-1 + i\sqrt{3})$" — remember the √3 is under i, not a^2. The three cube roots of unity are 1, ω, ω^2. They are equally spaced at 120° apart on the unit circle.

4.2

Properties of ω — The Most Tested Results

Memorise ALL properties — they are used across many NDA questions

⚡ **CORE PROPERTIES OF CUBE ROOTS OF UNITY**

P1. $\omega^3 = 1$ (and by extension $\omega^6 = 1$, $\omega^9 = 1$, ...)

P2. $1 + \omega + \omega^2 = 0$ ← most important

P3. $\omega^2 = 1/\omega$ (equivalently: $\omega \cdot \omega^2 = \omega^3 = 1$)

P4. $\bar{\omega} = \omega^2$ (they are conjugates)

P5. $|\omega| = |\omega^2| = 1$ (both lie on unit circle)

P6. $(\omega)^{(3k)} = 1$ for any integer k

P7. ω^n repeats with period 3: check n mod 3

P2 is the single most tested property. Every NDA set on ω eventually uses $1+\omega+\omega^2=0$ somewhere in the solution.

Derived Results

- $\omega + \omega^2 = -1$
- $\omega \cdot \omega^2 = 1$ (product)
- $(1+\omega) = -\omega^2$
- $(1+\omega^2) = -\omega$
- $\omega^2 + 1/\omega = 0$

Powers of ω (cycle 3)

- $\omega^0 = 1,\ \omega^1 = \omega,\ \omega^2 = \omega^2$
- $\omega^3 = 1,\ \omega^4 = \omega,\ \omega^5 = \omega^2$
- ω^n → find n mod 3
- mod 3 = 0 → 1
- mod 3 = 1 → ω
- mod 3 = 2 → ω^2

Key Expressions

- $(1+\omega)^3 = (-\omega^2)^3 = -\omega^6 = -1$
- $(1+\omega^2)^3 = (-\omega)^3 = -\omega^3 = -1$
- $\omega^2 + \omega + 1 = 0$ (same as P2)
- $a+b+c=0 \implies a^3+b^3+c^3=3abc$
- $1^3+\omega^3+(\omega^2)^3 = 3\cdot1\cdot\omega\cdot\omega^2 = 3$

Argand Diagram — Three Cube Roots of Unity

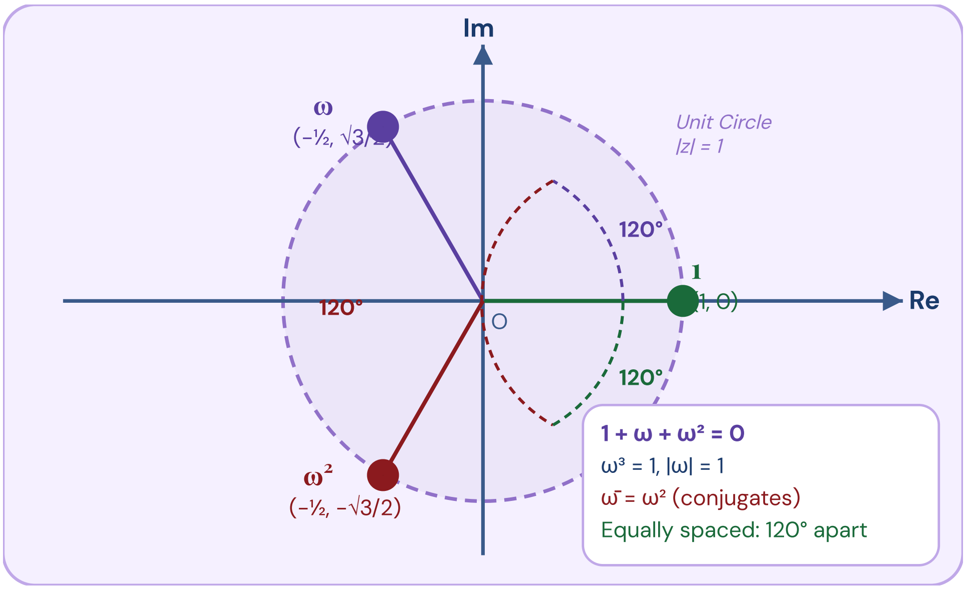

Fig 2: Argand Plane — Three cube roots of unity (1, ω, ω²) on the unit circle, each 120° apart. The centroid of the triangle they form is the origin.

TOPIC-WISE PYQ

Cube Roots of Unity — NDA-Pattern Questions

Q9. If ω is a complex cube root of unity, then $1 + \omega + \omega^2 = ?$

(a) 1 (b) −1 (c) 0 (d) i

Answer: (c) 0
This is the fundamental property. $1+\omega+\omega^2 = 0$ always, when ω is a non-real cube root of unity.

Q10. The value of $(1 + \omega)^3$ is:

(a) 1 (b) ω (c) $-\omega^2$ (d) −1

Answer: (d) −1
Since $1+\omega+\omega^2=0$, we get $1+\omega = -\omega^2$. So $(1+\omega)^3 = (-\omega^2)^3 = -(\omega^2)^3 = -(\omega^3)^2 = -(1)^2 =$ **−1**.

Q11. If ω is a cube root of unity, then ω^100 + ω^200 + ω^300 = ?

(a) −1 (b) 0 (c) 3 (d) 1

Answer: (c) 3
100 = 3×33+1 → ω^100 = ω^1 = ω.   200 = 3×66+2 → ω^200 = ω^2.   300 = 3×100 → ω^300 = ω^0 = 1.
Wait — re-check: 300 ÷ 3 = 100 exactly, remainder 0 → ω^300 = (ω^3)^100 = 1^100 = 1. Hmm — but also ω^100 = ω^1 and ω^200 = ω^2. So sum = $\omega + \omega^2 + 1 =$ **0**. (Re-check option: (b) 0 is the answer.)
Corrected Answer: (b) 0. Sum = $\omega + \omega^2 + 1 = 0$ by the fundamental property.

Q12. If 1, ω, ω^2 are cube roots of unity, the value of $(2+3\omega+2\omega^2)^2$ is:

(a) 9ω (b) $9\omega^2$ (c) ω (d) ω^2

Answer: (b) $9\omega^2$
Use $1+\omega+\omega^2=0 \rightarrow \omega+\omega^2=-1$. Rewrite: $2+3\omega+2\omega^2 = 2(1+\omega^2)+3\omega = 2(-\omega)+3\omega$ [since $1+\omega^2=-\omega$] $= -2\omega+3\omega =$ **ω**. Hmm — actually: $2+3\omega+2\omega^2 = 2+2\omega^2+3\omega = 2(1+\omega^2)+3\omega$. Since $1+\omega^2 = -\omega \rightarrow 2(-\omega)+3\omega = \omega$. So $(\omega)^2 =$ **ω^2**. Answer: **(d) ω^2**.
Strategy: Always group terms using $1+\omega+\omega^2=0$ to simplify the bracket before squaring.

🔥 TRICKY QUESTIONS

Cube Roots of Unity — High-Value Traps

🧩 **T6. Show that $1^3 + \omega^3 + (\omega^2)^3 = 3$. But what is $1 + \omega^3 + \omega^6$?**

$1 + \omega^3 + \omega^6 = 1 + 1 + 1 = 3$.
$\omega^3 = 1$ and $\omega^6 = (\omega^3)^2 = 1^2 = 1$. So the sum = 3.
Trap: Students write $\omega^3 = \omega$ and $\omega^6 = \omega^2$. Wrong! $\omega^3 = 1$, not ω. The cube root cycle is 3, so any multiple of 3 in the exponent gives 1.

T7. If ω is a cube root of unity, evaluate $(1 - \omega + \omega^2)^4 + (1 + \omega - \omega^2)^4$.

Solution: -16.

Use $1+\omega+\omega^2=0 \rightarrow 1+\omega^2=-\omega$ and $1+\omega=-\omega^2$.

So: $1-\omega+\omega^2 = (1+\omega^2)-\omega = -\omega-\omega = -2\omega$.

And: $1+\omega-\omega^2 = (1+\omega)-\omega^2 = -\omega^2-\omega^2 = -2\omega^2$.

Sum = $(-2\omega)^4 + (-2\omega^2)^4 = 16\omega^4 + 16\omega^8 = 16\omega + 16\omega^2$ [since $\omega^4=\omega$, $\omega^8=\omega^2$]

= $16(\omega+\omega^2) = 16(-1) =$ **-16**.

T8. If α, β are complex cube roots of unity, what is αβ?

$\alpha\beta = \omega \cdot \omega^2 = \omega^3 = 1$.

The two non-real cube roots of unity are ω and ω^2. Their product is always 1 (since $\omega^3 = 1$).

Their sum $\omega + \omega^2 = -1$. These are the Vieta's formulas for $x^2 + x + 1 = 0$: sum of roots = −1, product of roots = 1.

Master Formula Sheet — MN02 Complex Numbers

All high-yield formulae in one place for rapid pre-exam revision.

Powers of Iota

- $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$ (cycle of 4)
- i^n: find remainder r = n mod 4, then i^r
- i^0 = 1 always
- i^(−1) = −i; i^(−2) = −1
- Sum of 4 consecutive powers = 0

Modulus

- $|a+ib| = \sqrt{a^2+b^2}$
- $|z_1z_2| = |z_1||z_2|$
- $|z|^2 = z\cdot\bar{z} = a^2+b^2$
- $|z_1+z_2| \le |z_1|+|z_2|$ (Triangle ineq.)
- $|z| = |\bar{z}|$ and $|z| \ge 0$

Conjugate

- $\bar{z} = a-ib$ if $z = a+ib$
- $z+\bar{z} = 2a = 2\mathrm{Re}(z)$

Argument

- arg(z) = θ = arctan(b/a) adjusted by quadrant

- ∴ $z-\bar{z} = 2ib$ (purely imaginary)
- ∴ $z\cdot\bar{z} = |z|^2$ (use in division)
- ∴ $z=\bar{z} \Leftrightarrow z$ real; $z=-\bar{z} \Leftrightarrow z$ imaginary

- ∴ arg(1)=0, arg(i)=π/2, arg(−1)=π, arg(−i)=−π/2
- ∴ $\arg(z_1z_2) = \arg(z_1)+\arg(z_2)$
- ∴ $\arg(z_1/z_2) = \arg(z_1)-\arg(z_2)$
- ∴ Principal value ∈ (−π, π]

🌀 Cube Roots of Unity

- ∴ $\omega = (-1+i\sqrt{3})/2$, $\omega^2 = (-1-i\sqrt{3})/2$
- ∴ $\omega^3 = 1$ and $1+\omega+\omega^2 = 0$ ← KEY
- ∴ $1+\omega = -\omega^2$ and $1+\omega^2 = -\omega$
- ∴ $\omega+\omega^2 = -1$, $\omega\cdot\omega^2 = 1$
- ∴ $\bar{\omega} = \omega^2$, $|\omega|=|\omega^2|=1$

√ Square Root of z

- ∴ Let $\sqrt{(a+ib)} = x+iy$
- ∴ $x^2 = (|z|+a)/2$, $y^2 = (|z|-a)/2$
- ∴ Sign of y: same as sign of b/x (i.e. 2xy=b)
- ∴ Two roots: ±(x+iy)
- ∴ Always verify by squaring back

⚡ Quick Revision Booster — MN02 Complex Numbers

🔢 Iota Shortcuts

- $i^{4k} = 1$ (any multiple of 4)
- $i^{(4k+1)} = i$
- $i^{(4k+2)} = -1$
- $i^{(4k+3)} = -i$
- Negative power: add 4 until positive
- Sum of 4 consecutive powers = 0

🎯 Modulus Must-Knows

- $|z|=0$ iff $z=0$
- $|z|^2 = z\cdot\bar{z}$ → use for division
- $|iz| = |i||z| = |z|$
- $|z^n| = |z|^n$
- Triangle ineq: $|z_1+z_2| \le |z_1|+|z_2|$
- Equal modulus ≠ equal complex numbers

🔄 Conjugate Rules

- Conjugate flips sign of Im part only
- z is real ⇔ $z = \bar{z}$
- z is imaginary ⇔ $z = -\bar{z}$
- $Re(z) = (z+\bar{z})/2$
- $Im(z) = (z-\bar{z})/2i$
- Multiply by conjugate to make denominator real

🌀 Omega (ω) Facts

📐 Argument Basics

🚨 Critical Exam

- $1+\omega+\omega^2=0$ ← most important
- $\omega^3=1$ (cycle of 3 for powers)
- $(1+\omega)=-\omega^2$, $(1+\omega^2)=-\omega$
- $(1+\omega)^3 = -1$
- ω^n: find n mod 3
- Three roots lie on unit circle at 120° gap

- $\arg(1)=0$, $\arg(i)=\pi/2$
- $\arg(-1)=\pi$, $\arg(-i)=-\pi/2$
- 1st Q: θ positive and acute
- 2nd Q: $\theta = \pi$ − basic angle
- 3rd Q: $\theta = -\pi$ + basic angle
- 4th Q: θ = −basic angle

Traps

- $i^2 = -1$, NOT +1
- $\sqrt{-a}\cdot\sqrt{-b} \neq \sqrt{ab}$; use i: $(i\sqrt{a})(i\sqrt{b}) = -\sqrt{ab}$
- $\omega^3=1$ NOT $\omega^3=\omega$ (power resets at 3)
- Im(z)=b is REAL (not ib)
- $|z_1|=|z_2|$ does NOT mean $z_1=z_2$
- $z+\bar{z}$ is purely real; $z-\bar{z}$ is purely imaginary

📝 Mock Tests 🎯 Subject Quizzes ✈️ Telegram