

Sets, Relations & Functions

Algebra · Chapter MN01

NDA Level : High Priority

Sets, Relations & Functions form the foundational language of all mathematics. In NDA Mathematics, this chapter directly contributes MCQs on counting elements of sets, applying set operations, classifying functions, and evaluating composite or inverse functions.

Questions are largely concept-based – no complex computation is needed, making this chapter one of the highest return-on-investment chapters for the exam.

✦ What to expect in NDA (based on 2022–2024 papers):

- (1) Finding $n(A \cup B)$ using the inclusion-exclusion formula;
- (2) Number of subsets / proper subsets of a set;
- (3) Venn diagram-based counting problems (3-set problems);
- (4) Verifying whether a relation is an equivalence relation;
- (5) Identifying the type of function (one-one, onto, bijective);
- (6) Evaluating composite functions (fog, gof);
- (7) Finding domain and range of simple functions;
- (8) De Morgan's Law application.

Topics at a Glance

① Sets & Notation

Roster form, set-builder form, types of sets

② Set Operations

Union, intersection, difference, complement

③ Venn Diagrams

2-set & 3-set counting, De Morgan's laws

④ Relations

Cartesian product, domain, range, types

⑤ Functions

One-one, onto, bijective, into, many-one

⑥ Composite & Inverse

fog, gof, f^{-1} , inverse conditions

1. Sets – Concepts & Notation

What is a Set? – Definition & Representations

A **set** is a well-defined collection of distinct objects (called **elements** or **members**). "Well-defined" means there is no ambiguity about whether an object belongs to the set.

Roster (Tabular) Form

- ▶ All elements are listed explicitly, separated by commas, inside curly braces $\{ \}$.
- ▶ Order doesn't matter; repetition is not allowed.
- ▶ Example: $A = \{1, 2, 3, 4, 5\}$ — set of first 5 natural numbers
- ▶ Example: $B = \{a, e, i, o, u\}$ — set of vowels
- ▶ Example: $C = \{2, 4, 6, 8, 10\}$ — even numbers up to 10

Set-Builder (Rule) Form

- ▶ Describes elements by a property: $A = \{x : \text{condition on } x\}$
- ▶ Read as "A is the set of all x such that..."
- ▶ Example: $A = \{x : x \in \mathbb{N}, x \leq 5\}$ — same as $\{1, 2, 3, 4, 5\}$
- ▶ Example: $B = \{x : x \text{ is a vowel in English alphabet}\}$
- ▶ Used when listing all elements is impractical (e.g., infinite sets)

⚠ Common Confusion: The set $\{0\}$ is NOT an empty set — it contains one element (zero). The empty set is $\{ \}$ or \emptyset . Also, sets $\{1, 2, 3\}$ and $\{3, 1, 2\}$ are identical — order does not matter in sets.

Types of Sets — Must-Know Classification

Directly tested in MCQs — identify the type from description

Type of Set	Definition	Example
Empty / Null Set (\emptyset)	Contains no elements	$\{x : x \in \mathbb{N}, x^2 = -1\}$
Singleton Set	Exactly one element	$\{5\}$ or $\{\emptyset\}$ (set containing empty set)
Finite Set	Countable number of elements	$\{2, 4, 6, 8, 10\}$
Infinite Set	Elements cannot be exhausted	$\mathbb{N} = \{1, 2, 3, \dots\}$, \mathbb{Z} , \mathbb{R}
Universal Set (U)	The overall set in context	Often $U = \mathbb{R}$ or a specific given set
Equal Sets	Same elements (order irrelevant)	$\{1, 2, 3\} = \{3, 2, 1\}$

Equivalent Sets	Same number of elements (not necessarily same)	$\{1,2,3\} \sim \{a,b,c\}$
Subset ($A \subseteq B$)	Every element of A is in B	$\{2,4\} \subseteq \{2,4,6\}$
Proper Subset ($A \subset B$)	$A \subseteq B$ but $A \neq B$	$\{2,4\} \subset \{2,4,6\}$
Power Set $P(A)$	Set of all subsets of A	If $A=\{a,b\}$ then $P(A)=\{\emptyset,\{a\},\{b\},\{a,b\}\}$
Disjoint Sets	No common elements; $A \cap B = \emptyset$	$\{1,3\}$ and $\{2,4\}$

⚡ **POWER SET FORMULA (HIGH FREQUENCY IN NDA)**

If $n(A) = n$, then:

Number of subsets = 2^n

Number of proper subsets = $2^n - 1$

Number of non-empty proper subsets = $2^n - 2$

If A has 3 elements: subsets = $2^3 = 8$; proper subsets = 7; $P(A)$ has 8 elements.

2. Set Operations

2.1

Union, Intersection, Difference & Complement

Core operations — always paired with Venn diagram questions

Union ($A \cup B$)

- ▶ All elements belonging to A or B (or both)
- ▶ If $A=\{1,2,3\}$ and $B=\{3,4,5\}$, then $A \cup B = \{1,2,3,4,5\}$
- ▶ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- ▶ $A \cup \emptyset = A$; $A \cup U = U$; $A \cup A = A$
- ▶ $A \cup A' = U$ (complement law)

Intersection ($A \cap B$)

- ▶ Elements common to both A and B
- ▶ If $A=\{1,2,3\}$ and $B=\{3,4,5\}$, then $A \cap B = \{3\}$
- ▶ $A \cap \emptyset = \emptyset$; $A \cap U = A$; $A \cap A = A$
- ▶ $A \cap A' = \emptyset$ (complement law)
- ▶ If $A \cap B = \emptyset$, sets are disjoint

Difference (A - B or A \ B)

- ▶ Elements in A but NOT in B
- ▶ If $A=\{1,2,3\}$ and $B=\{3,4,5\}$, then $A-B=\{1,2\}$
- ▶ $A - B \neq B - A$ in general
- ▶ $A - B = A \cap B'$ (complement of B)
- ▶ $n(A-B) = n(A) - n(A \cap B)$

Complement (A')

- ▶ All elements of Universal Set U that are NOT in A
- ▶ $A' = U - A$
- ▶ $(A')' = A$ (double complement)
- ▶ $U' = \emptyset$; $\emptyset' = U$
- ▶ $n(A') = n(U) - n(A)$

⚡ SYMMETRIC DIFFERENCE (OFTEN ASKED AS "WHICH OPERATION IS THIS?")

$$A \Delta B = (A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

Contains elements in A or B but NOT in both. If $A=\{1,2,3\}$, $B=\{2,3,4\} \rightarrow A \Delta B = \{1,4\}$

2.2

Venn Diagrams & Counting Formulae

The most tested application – 3-set problems appear almost every year

Two-Set Venn Diagram

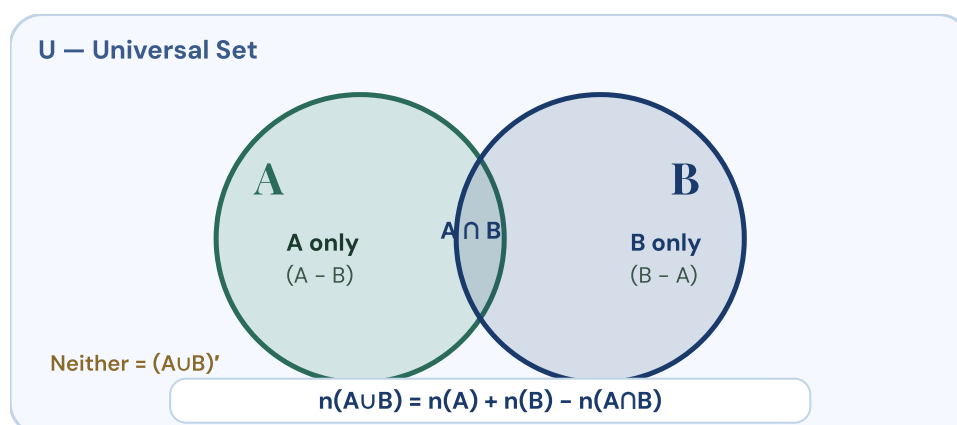
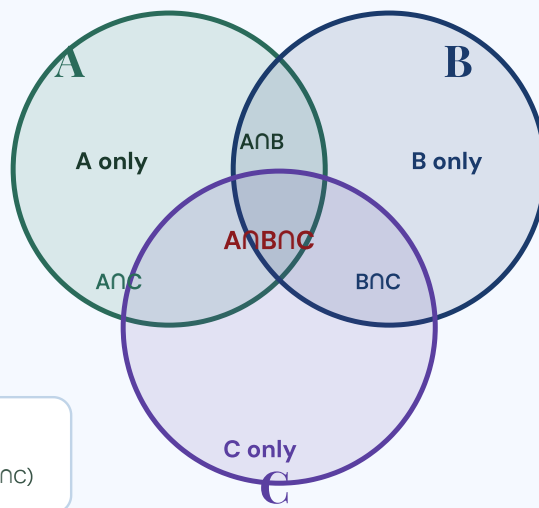


Fig 1: Two-Set Venn Diagram – shaded regions indicate set operations

Three-Set Venn Diagram (NDA Favourite)

U – Universal Set



$$\begin{aligned} n(A \cup B \cup C) = & n(A) + n(B) + n(C) \\ & - n(A \cap B) - n(B \cap C) - n(A \cap C) \\ & + n(A \cap B \cap C) \end{aligned}$$

Fig 2: Three-Set Venn Diagram – 7 distinct regions shown; used in survey-type word problems

⚡ KEY COUNTING FORMULAE – SET THEORY

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$n(\text{A only}) = n(A) - n(A \cap B) \text{ [for 2 sets]}$$

$$n(\text{exactly one of A or B}) = n(A) + n(B) - 2 \cdot n(A \cap B)$$

In 3-set problems: $n(\text{exactly one}) = n(A) + n(B) + n(C) - 2[n(A \cap B) + n(B \cap C) + n(A \cap C)] + 3 \cdot n(A \cap B \cap C)$

2.3

De Morgan's Laws & Important Properties

Statement-based questions – know both forms cold

⚡ DE MORGAN'S LAWS

$$\text{Law 1: } (A \cup B)' = A' \cap B'$$

$$\text{Law 2: } (A \cap B)' = A' \cup B'$$

In plain words – Law 1: Complement of a union = intersection of complements. Law 2: Complement of an intersection = union of complements.

Property	For Union (\cup)	For Intersection (\cap)
Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative	$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cap (B \cap C) = (A \cap B) \cap C$
Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity	$A \cup \emptyset = A$	$A \cap U = A$
Idempotent	$A \cup A = A$	$A \cap A = A$
Complement	$A \cup A' = U$	$A \cap A' = \emptyset$

✦ **Memory Aid for De Morgan's:** "Break the bar, change the operation."
 Complement of $(A \cup B) \rightarrow$ break bar to get A' and B' , change \cup to \cap .

TOPIC-WISE PYQ

Sets — NDA-Pattern Questions

Q1. In a class of 50 students, 30 like Mathematics, 25 like Science, and 10 like both. How many students like neither?

- (a) 5 (b) 10 (c) 15 (d) 45

Answer: (a) 5

$n(M \cup S) = n(M) + n(S) - n(M \cap S) = 30 + 25 - 10 = 45$. Students who like neither = $50 - 45 = 5$.

Q2. If $A = \{1, 2, 3\}$ then the number of proper subsets of A is:

- (a) 6 (b) 7 (c) 8 (d) 9

Answer: (b) 7

Number of subsets = $2^3 = 8$. Proper subsets = $8 - 1 = 7$ (excluding A itself).

Q3. If $n(A) = 4$, $n(B) = 5$ and $n(A \cap B) = 3$, then $n(A \cup B)$ is:

- (a) 4 (b) 5 (c) 6 (d) 7

Answer: (c) 6

$$n(A \cup B) = 4 + 5 - 3 = 6.$$

Q4. Which of the following is De Morgan's Law?

(a) $(A \cup B)' = A' \cup B'$ (b) $(A \cap B)' = A' \cap B'$ (c) $(A \cup B)' = A' \cap B'$ (d) $A \cup B = A \cap B$

Answer: (c)

De Morgan's First Law: $(A \cup B)' = A' \cap B'$. Option (a) and (b) are both incorrect forms.



TRICKY QUESTIONS

Sets – Common Traps & Their Solutions

✿ T1. In a survey of 100 people: 70 like tea, 60 like coffee, and some like both. What is the minimum number who must like both?

Solution: $n(T \cup C) \leq 100$ (cannot exceed total). So $n(T) + n(C) - n(T \cap C) \leq 100 \rightarrow 70 + 60 - n(T \cap C) \leq 100 \rightarrow n(T \cap C) \geq 30$. **Minimum = 30.**

Trap: Students often forget the constraint $n(A \cup B) \leq n(U)$.

✿ T2. If $A \subseteq B$, then $A \cap B = ?$

Solution: $A \cap B = A$. If every element of A is already in B, then the common elements are just all of A. Similarly, $A \cup B = B$ in this case.

Trap: Students often say $A \cap B = B$. Remember: intersection takes the smaller set when one is a subset of the other.

✿ T3. In a group of 100 students: 40 study Hindi, 50 study English, 20 study Sanskrit, 15 study Hindi and English, 10 study English and Sanskrit, 8 study Hindi and Sanskrit, and 5 study all three. How many study none?

Solution: $n(H \cup E \cup S) = 40 + 50 + 20 - 15 - 10 - 8 + 5 = 110 - 33 + 5 = 82$. Students who study none = $100 - 82 = 18$.

3. Relations

3.1

Cartesian Product & Definition of a Relation

Foundation of all relation theory

The **Cartesian product** $A \times B$ is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. A **relation R** from A to B is any subset of $A \times B$.

⚡ CARTESIAN PRODUCT KEY FACTS

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

$$n(A \times B) = n(A) \times n(B)$$

If $n(A) = m$ and $n(B) = n$, then number of relations from A to B = $2^{(mn)}$

Note: $A \times B \neq B \times A$ in general (ordered pairs — order matters). $A \times B = B \times A$ only if $A = B$ or one of them is empty.

Domain of Relation R

- ▶ Set of all first elements (x-values) in the ordered pairs of R
- ▶ Domain \subseteq Set A (the first set)
- ▶ Example: If $R = \{(1,2), (1,3), (2,4)\}$,
Domain = $\{1,2\}$

Range of Relation R

- ▶ Set of all second elements (y-values) in the ordered pairs of R
- ▶ Range \subseteq Set B (the second set)
- ▶ Example: If $R = \{(1,2), (1,3), (2,4)\}$,
Range = $\{2,3,4\}$

⚠ Domain vs Co-domain: Co-domain is the entire set B (the "target" set). Range is only the set of values actually attained. Range \subseteq Co-domain always. They are equal only when the relation is onto (surjective).

3.2

Types of Relations — Reflexive, Symmetric, Transitive & Equivalence

Tested by definition + verification — very common in NDA

Type	Definition	Test Condition	Example
Reflexive	Every element is related to itself	$(a, a) \in R$ for all $a \in A$	$R = \{(1,1), (2,2), (3,3), \dots\}$ on \mathbb{N}
Irreflexive	No element is related to itself	$(a, a) \notin R$ for all $a \in A$	$R =$ "is strictly less than" on \mathbb{N}
Symmetric	If aRb then bRa	$(a,b) \in R \Rightarrow (b,a) \in R$	$R =$ "is a sibling of" (mutual)
Anti-symmetric	If aRb and bRa then $a = b$	$(a,b) \in R$ and $(b,a) \in R \Rightarrow a=b$	$R =$ " \leq " on real numbers
Transitive	If aRb and bRc then aRc	$(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$	$R =$ "is ancestor of"
Equivalence	Reflexive + Symmetric + Transitive	All three must hold simultaneously	$R =$ "has same age as", " $\equiv \pmod{n}$ "

✦ NDA Shortcut – Checking Equivalence Relation:

Always check all THREE: (1) Is (a,a) always there? [Reflexive] \rightarrow (2) If (a,b) is there, is (b,a) always there? [Symmetric] \rightarrow (3) If (a,b) and (b,c) are there, is (a,c) always there? [Transitive]. Only if all three pass \rightarrow Equivalence Relation.

Important Relations to Know:

- " $=$ " (equality) on any set – always an equivalence relation.
- " \leq " (less than or equal to) – reflexive, transitive, anti-symmetric but NOT symmetric \rightarrow NOT equivalence.
- " \subseteq " (subset) – reflexive, transitive, anti-symmetric \rightarrow NOT equivalence.
- " \parallel " (parallel lines) – equivalence relation.
- " $\equiv \pmod{n}$ " (congruence modulo n) – equivalence relation (most important example).

- (a) 5 (b) 6 (c) 8 (d) 9

Answer: (b) 6

$$n(A \times B) = n(A) \times n(B) = 3 \times 2 = 6.$$

Q6. A relation R on set A = {1, 2, 3} is defined as R = {(1,1),(2,2),(3,3),(1,2),(2,1)}.

Which of the following is correct?

- (a) Reflexive only (b) Symmetric only (c) Reflexive and Symmetric only (d) Equivalence relation

Answer: (c) Reflexive and Symmetric only

Reflexive: (1,1),(2,2),(3,3) ✓. Symmetric: (1,2) → (2,1) ✓. Transitive: (1,2) and (2,1) → need (1,1) ✓; BUT check all pairs — satisfied here. Actually it IS transitive. **Equivalence relation** ✓.

Note: Always check carefully. If (2,1) and (1,2) are both in R, you need (2,2) for transitivity → (2,2) is present ✓.

Q7. Which of the following is NOT an equivalence relation?

- (a) "Is equal to" on \mathbb{R} (b) "Is congruent to" on triangles (c) "Is less than" on \mathbb{R} (d) "Is parallel to" on lines in a plane

Answer: (c) "Is less than"

"Less than" is not reflexive ($a < a$ is false) and not symmetric ($a < b$ does not mean $b < a$). So it fails to be an equivalence relation.

4. Functions — Types & Classification

4.1

What is a Function? — Domain, Co-domain, Range

Every function is a relation, but not every relation is a function

A function $f: A \rightarrow B$ is a special relation in which every element of A (domain) is related to exactly *one* element of B (co-domain). The set B is the co-domain; the actual set of mapped values is the range.

✦ The Two Rules of a Function:

Rule 1: Every element in the domain must have an image (no element left unmapped).

Rule 2: Each element in the domain must have exactly ONE image (no element maps to two or more outputs).

If either rule is violated → it is a relation but NOT a function.

Visual Summary — Is it a Function?

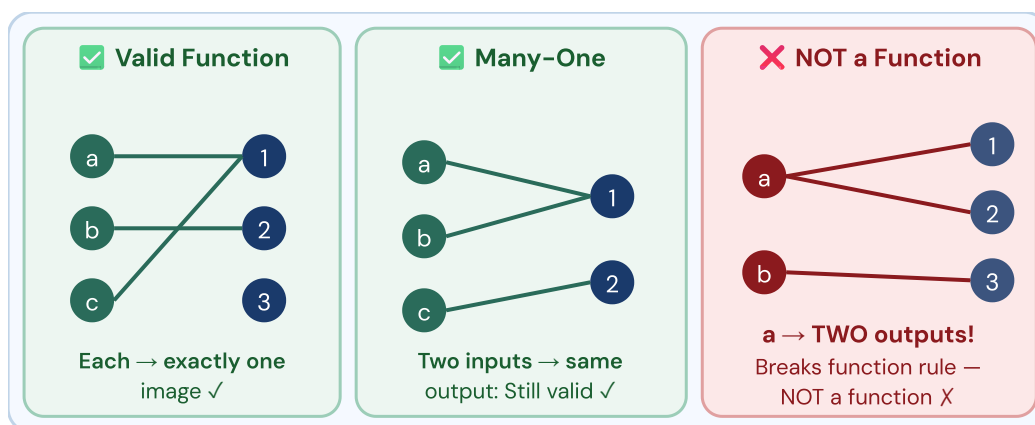


Fig 3: Function mapping diagrams — identifying valid vs invalid functions

Types of Functions — One-One, Onto, Bijective, Many-One, Into

4.2

Most MCQs ask you to identify the type — master the definitions

Type	Other Name	Definition	Range vs Co-domain	Example
One-One	Injective	Different inputs give different outputs. If $f(a) = f(b) \Rightarrow a = b$	Range \subseteq Co-domain	$f(x)=2x$ on $\mathbb{R} \rightarrow \mathbb{R}$
Many-One	—	At least two different inputs map to the same output	Range \subseteq Co-domain	$f(x)=x^2$ on \mathbb{R} : $f(2)=f(-2)=4$
Onto	Surjective	Every element in co-domain B has at least one pre-image in A	Range = Co-domain	$f(x)=x^3$ on $\mathbb{R} \rightarrow \mathbb{R}$
Into	—	At least one element in B has no pre-image; range is strict subset of B	Range \subset Co-domain	$f(x)=x^2$ from $\mathbb{R} \rightarrow \mathbb{R}$ (no negative output)
Bijective	One-One + Onto	Every element of B is mapped from exactly one	Range = Co-domain	$f(x)=x+1$ on $\mathbb{R} \rightarrow \mathbb{R}$

⚡ QUICK DECISION CHART – IDENTIFYING FUNCTION TYPE

Step 1: Is it one-one? → Check if $f(a)=f(b)$ always means $a=b$

Step 2: Is it onto? → Check if Range = Co-domain (every y has an x)

→ **One-one + Onto = Bijective (Invertible)**

→ **One-one + Into = Injective but not surjective**

→ **Many-one + Onto = Surjective but not injective**

→ **Many-one + Into = Neither**

For finite sets: Bijection exists only if $n(A) = n(B)$. Injective: $n(A) \leq n(B)$. Surjective: $n(A) \geq n(B)$.

⚠ Key Examples to Memorise:

- $f(x) = x^2$ on $\mathbb{R} \rightarrow \mathbb{R}$: **Many-One, Into** ($f(2)=f(-2)$; negative reals not in range)
- $f(x) = x^2$ on $\mathbb{R} \rightarrow [0, \infty)$: **Many-One, Onto** (range now equals co-domain)
- $f(x) = x^2$ on $[0, \infty) \rightarrow [0, \infty)$: **One-One, Onto = Bijective**
- $f(x) = \sin x$ on $\mathbb{R} \rightarrow \mathbb{R}$: **Many-One, Into** (range = $[-1, 1] \neq \mathbb{R}$)
- $f(x) = e^x$ on $\mathbb{R} \rightarrow \mathbb{R}$: **One-One, Into** (all outputs > 0)

4.3

Composite Functions & Inverse Functions

fog vs gof – order matters! Very common in NDA MCQs

Composite Function (fog)

- ▶ $(fog)(x) = f(g(x))$ – apply g first, then f
- ▶ $fog \neq gof$ in general (NOT commutative)
- ▶ fog is defined when range of $g \subseteq$ domain of f
- ▶ $(fog)oh = fo(goh)$ – Associative
- ▶ $f \circ f^{-1} = f^{-1} \circ f = I$ (Identity function)

Inverse Function (f^{-1})

- ▶ Exists only if f is **bijective** (one-one and onto)
- ▶ $f^{-1} : B \rightarrow A$ reverses the mapping
- ▶ Domain of $f^{-1} =$ Range of f
- ▶ Range of $f^{-1} =$ Domain of f
- ▶ $(f^{-1})^{-1} = f$

WORKED EXAMPLE – COMPOSITE FUNCTIONS

Let $f(x) = 2x + 1$ and $g(x) = x^2$. Find $fog(x)$ and $gof(x)$.

$$f \circ g(x) = f(g(x)) = f(x^2) = 2(x^2) + 1 = 2x^2 + 1$$

$$g \circ f(x) = g(f(x)) = g(2x+1) = (2x+1)^2 = 4x^2 + 4x + 1$$

$f \circ g(x) \neq g \circ f(x) \rightarrow$ confirms composition is NOT commutative.

WORKED EXAMPLE – INVERSE FUNCTION

Find f^{-1} if $f(x) = 3x - 5$ (f is bijective on $\mathbb{R} \rightarrow \mathbb{R}$).

Step 1: Let $y = f(x) = 3x - 5$. Step 2: Solve for x : $x = (y + 5)/3$. Step 3: So $f^{-1}(y) = (y + 5)/3$ or written as $f^{-1}(x) = (x + 5)/3$.

TOPIC-WISE PYQ

Functions — NDA–Pattern Questions

Q8. $f(x) = x^2$ defined from \mathbb{R} to \mathbb{R} is:

- (a) One-one and onto (b) Many-one and into (c) One-one and into (d) Many-one and onto

Answer: (b) Many-one and into

$f(2) = f(-2) = 4 \rightarrow$ Many-one. Negative reals (e.g., -1) have no pre-image \rightarrow Range = $[0, \infty) \neq \mathbb{R} \rightarrow$ Into.

Q9. If $f(x) = 2x + 3$ and $g(x) = (x - 3)/2$, then $f \circ g(x) = ?$

- (a) $x - 3$ (b) x (c) $2x + 3$ (d) $x + 3$

Answer: (b) x

$f \circ g(x) = f(g(x)) = f((x-3)/2) = 2 \cdot (x-3)/2 + 3 = (x-3) + 3 = x$. This shows $g = f^{-1}$ (they are inverse functions).

Q10. A function $f: A \rightarrow B$ is invertible if and only if f is:

- (a) One-one only (b) Onto only (c) Bijective (d) Into

Answer: (c) Bijective

An inverse function exists if and only if f is both one-one (injective) AND onto (surjective), i.e., bijective.

Q11. The number of functions from a set A with 3 elements to a set B with 2 elements is:

- (a) 6 (b) 8 (c) 9 (d) 12

Answer: (b) 8

Each of the 3 elements of A can map to any of the 2 elements of B. Total functions = $2^3 = 8$.

Note: This is different from $n(A \times B) = 6$. "Number of functions" = $[n(B)]^{n(A)}$.



TRICKY QUESTIONS

Functions — Common Traps & Their Solutions

✿ T4. $f(x) = |x|$ (absolute value) from $\mathbb{R} \rightarrow \mathbb{R}$. What type of function is it?

Solution: Many-One, Into.

Many-One: $f(3) = f(-3) = 3 \rightarrow$ two inputs give same output.

Into: Range = $[0, \infty) \neq \mathbb{R}$ (co-domain) \rightarrow negative values are not achieved.

Trap: Students often call it "one-one" because $|x|$ looks simple. Always check with negative inputs.

✿ T5. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = x^3$, is it bijective?

Solution: Yes, Bijective.

One-One: If $x_1^3 = x_2^3 \rightarrow x_1 = x_2 \checkmark$ (cube function is strictly increasing).

Onto: Every real number y has exactly one real cube root $x = y^{1/3} \checkmark$.

$\rightarrow f$ is bijective and $f^{-1}(x) = x^{1/3}$.

Contrast with x^2 which is NOT bijective on \mathbb{R} .

✿ T6. If $f(x) = x + 1$ and $g(x) = x - 1$, find $(f \circ g)(5)$ and $(g \circ f)(5)$.

$f \circ g(5) = f(g(5)) = f(5-1) = f(4) = 4+1 = 5$.

$g \circ f(5) = g(f(5)) = g(5+1) = g(6) = 6-1 = 5$.

Here $f \circ g = g \circ f = x$ (identity). This happens because f and g are inverse functions of each other.

Trap: Students assume $f \circ g \neq g \circ f$ always. It can be equal in special cases (like inverse pairs).

✿ T7. How many bijective functions exist from $A = \{1,2,3\}$ to $B = \{a,b,c\}$?

Solution: $3! = 6$.

A bijective function from a set of n elements to another set of n elements is a permutation. Number of bijections = $n! = 3! = 6$.

Key Formula: Bijections from A to B (n elements each) = $n!$

Master Formula Sheet – MNo1 Algebra

Use this for rapid revision before exam. All high-yield formulae compiled in one place.

Set Counting

- $\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $\therefore n(A - B) = n(A) - n(A \cap B)$
- $\therefore n(A') = n(U) - n(A)$
- $\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- \therefore Exactly one of $A, B = n(A) + n(B) - 2 \cdot n(A \cap B)$

Subsets & Power Set

- \therefore Total subsets of $A = 2^n(A)$
- \therefore Proper subsets = $2^n - 1$
- \therefore Non-empty proper subsets = $2^n - 2$
- $\therefore |P(A)| = 2^n$ (power set has 2^n elements)
- $\therefore \emptyset$ and A itself are always subsets of A

Cartesian Product

- $\therefore n(A \times B) = n(A) \times n(B)$
- $\therefore A \times B \neq B \times A$ (unless $A=B$ or either is \emptyset)
- \therefore Number of relations from A to $B = 2^{(mn)}$
- \therefore Number of functions from A to $B = n(B)^{n(A)}$
- \therefore Number of bijections = $n!$ (if $n(A)=n(B)=n$)

De Morgan's & Laws

- $\therefore (A \cup B)' = A' \cap B'$
- $\therefore (A \cap B)' = A' \cup B'$
- $\therefore (A')' = A$
- $\therefore A \Delta B = (A \cup B) - (A \cap B)$
- $\therefore A - B = A \cap B'$

Equivalence Relation

- \therefore Must be: Reflexive + Symmetric + Transitive

Functions Quick-Ref

- $\therefore fog(x) = f(g(x))$ – apply g first

- ∴ "=" on any set: equivalence ✓
- ∴ "≡ mod n" on \mathbb{Z} : equivalence ✓
- ∴ "||" (parallel) on lines: equivalence ✓
- ∴ "<" (less than) on \mathbb{R} : NOT equivalence
X

- ∴ Inverse exists \Leftrightarrow f is bijective
- ∴ $\text{Domain}(f^{-1}) = \text{Range}(f)$
- ∴ $f \circ f^{-1} = f^{-1} \circ f = \text{Identity } I(x)=x$
- ∴ $f(x)=x^2, f(x)=|x|, f(x)=\sin x \rightarrow$ many-one

⚡ Quick Revision Booster — MNo1 Sets, Relations & Functions

📦 Sets — Key Facts

- Empty set \emptyset is subset of every set
- Every set is subset of itself
- Power set of $\emptyset = \{\emptyset\}$ (has 1 element)
- Disjoint sets: $A \cap B = \emptyset$
- If $A \subseteq B$: $A \cap B = A$ and $A \cup B = B$
- Symmetric diff: $A \Delta B = A \cup B - A \cap B$

📊 Counting Traps

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B)$ max when $A \cap B = \emptyset$: $\text{max} = n(A) + n(B)$
- $n(A \cap B)$ min = $\text{max}(0, n(A) + n(B) - n(U))$
- 2^n subsets; $2^n - 1$ proper subsets
- Number of functions $A \rightarrow B = n(B)^{n(A)}$
- Bijections = $n!$ when $|A| = |B| = n$

🔄 De Morgan's

Rules

- $(A \cup B)' = A' \cap B'$ (flip union to intersection)
- $(A \cap B)' = A' \cup B'$ (flip intersection to union)
- Applied in logic: NOT(P OR Q) = NOT P AND NOT Q
- Double complement: $(A')' = A$
- Verify: always check with examples

🔗 Relations — Memory Aid

- Reflexive: "Everyone loves themselves" $\rightarrow (a,a) \in R$
- Symmetric: "If I like you, you like me" $\rightarrow (a,b) \Rightarrow (b,a)$
- Transitive: "Friend of friend is friend" \rightarrow chain holds

⚡ Function Classification

- One-one: no two x give same y
- Onto: range = co-domain
- Into: range \subset co-domain (strict)
- Bijective = one-one + onto \rightarrow invertible

🚨 Critical Exam Traps

- $\{0\} \neq \emptyset$ — set containing zero is NOT empty
- \emptyset has 1 subset (itself), so $2^0 = 1$ ✓
- $f \circ g \neq g \circ f$ in general (order matters)

- Equivalence = all 3 combined
- "<" fails reflexivity → not equivalence

- x^2 is many-one; x^3 is one-one
- $\sin x$, $\cos x$, $|x|$ are all many-one

- Inverse exists only for bijective functions
- Every equivalence relation is reflexive — check this first
- $\text{Range} \subseteq \text{Co-domain}$ (never the other way)



Mock Tests



Subject Quizzes



Telegram

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