Percentage, Ratio & Proportion

📐 Arithmetic · MC02 | CDS Elementary Mathematics | 🎯 High Priority

Percentage, Ratio and Proportion are the **workhorses of CDS Arithmetic**. Every application chapter — Profit & Loss, SI/CI, Time & Work — borrows these tools. Master the conversion triangle, the successive-change formula, and the proportion rules, and you gain speed across the entire paper.

📌 **CDS exam focus (recent papers):** (1) % increase/decrease and its reverse; (2) Successive % change — the shortcut formula saves 40 seconds per question; (3) Election problems — votes, margins, total voters; (4) Ratio comparison and compound ratios; (5) Proportion word problems — direct, inverse, continued; (6) Partnership — profit sharing when capitals and time differ; (7) Componendo-Dividendo applied to simplify ratio equations.

Topics at a Glance

① Percentage Basics
Conversion, % of, % change, reverse %

② Successive % Change
(a+b+ab/100)% — the key shortcut

③ Applications
Election, population, income/expenditure

④ Ratio & Comparison
Simplification, compound, duplicate ratios

⑤ Proportion Types
Direct, inverse, continued, mean proportion

⑥ Partnership
Capital × time ratio → profit share

1. Percentage — Core Concepts

1.1

The Conversion Triangle & Percentage Formulas

Every percentage problem is one of five standard types

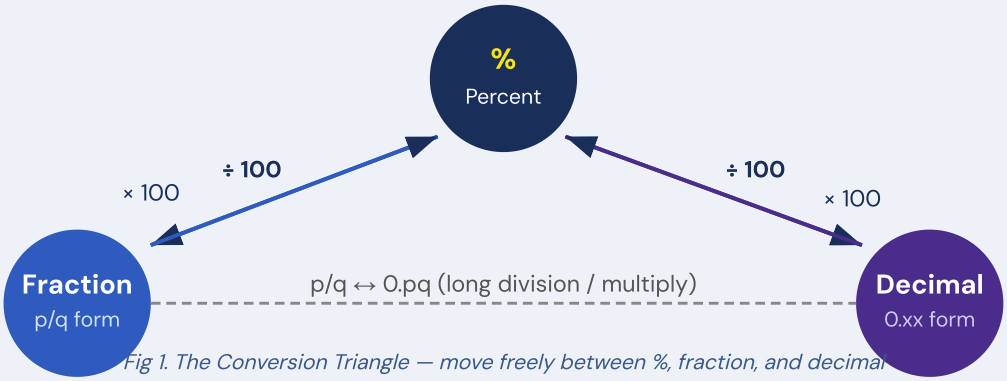

Fig 1. The Conversion Triangle — move freely between %, fraction, and decimal

THE FIVE STANDARD PERCENTAGE FORMULAS

Type 1 — What % is A of B?

Answer = (A / B) × 100

Type 2 — What is x% of A?

Answer = (x / 100) × A

Type 3 — % Increase / Decrease:

% Change = [(New − Old) / Old] × 100

New = Old × (1 ± x/100)

Type 4 — Reverse % (find original from changed value):

Original = New Value × 100 / (100 + x%) [if increased]

Original = New Value × 100 / (100 − x%) [if decreased]

Type 5 — Successive % change (a% then b%):

Net % change = a + b + (ab/100) [+ = increase, − = decrease]

Type 5 is the most tested shortcut in CDS. Use it whenever two consecutive percentage changes apply to the same base.

WORKED EXAMPLE — SUCCESSIVE % CHANGE

A salary is increased by 20% then decreased by 15%. What is the net % change?

Net = 20 + (−15) + (20 × −15)/100 = 5 − 3 = +2% (net increase of 2%).

Check: 100 → ×1.20 → 120 → ×0.85 → 102. Change = 2% ✓

A price rises by 10% then again by 10%. Net increase?

Net = 10 + 10 + (10×10)/100 = 20 + 1 = **21%**. (Not 20% — the second 10% is on a higher base.)

1.2 Election & Population Problems

Standard CDS application — 1–2 questions per paper

🗳️ Election Problems — Template

- Two candidates: Winner gets W%, Loser gets (100-W)%
- Majority (winning margin) = (W - (100-W))% of total votes
- **Margin = (2W - 100)% × Total Votes**
- Total Votes = Margin / (% difference / 100)
- Invalid votes: subtract first, then apply % to valid votes
- Example: Wins by 15%, margin = 4500 → Total = 4500/0.15 = 30,000

👥 Population Growth/Depreciation

- Population after n years: $P(1 + r/100)^n$
- Population n years ago: Present ÷ $(1 + r/100)^n$
- With different rates each year: $P(1+r_1/100)(1+r_2/100)...$
- Depreciation: $V(1 - r/100)^n$
- CDS often gives present population and asks past or future value
- Read carefully: "increases BY 5%" vs "increases TO 5%"

📋 TOPIC-WISE PYQ **Percentage — CDS Questions**

Q1. If the price of petrol increases by 25% and a person wants to spend only 15% more on petrol, by what % must he reduce consumption?

(a) 6% (b) 8% (c) 10% (d) 12%

Answer: (b) 8%

Let original: price P, qty Q, expenditure PQ. New price = 1.25P, new expenditure =

1.15PQ.
New qty = 1.15PQ / 1.25P = 0.92Q. Reduction = (1 − 0.92) × 100 = **8%**.

Q2. In an election between two candidates, 80% of voters cast votes. The winner got 55% of the votes polled and won by 1200 votes. Find the total number of voters on the list.

(a) 12,000 (b) 15,000 (c) 20,000 (d) 25,000

Answer: (b) 15,000
Votes polled = 80% of total. Winner: 55%, Loser: 45% of votes polled. Margin = 10% of votes polled = 1200. Votes polled = 12,000. Total = 12,000/0.8 = **15,000**.

Q3. A number is first increased by 20% and then decreased by 20%. The net change in the number is:

(a) 0% (b) −4% (c) +4% (d) −2%

Answer: (b) −4%
Successive change: 20 + (−20) + (20)(−20)/100 = 0 − 4 = **−4%**. The number decreases by 4%.

2. Ratio & Comparison

2.1 Types of Ratios & Comparison Techniques

Ratio appears directly and inside proportion, partnership, and mixture problems

A ratio a:b means for every a units of the first quantity, there are b units of the second. The ratio a:b is the same as the fraction a/b.

Types of Ratio

- **Duplicate ratio** of a:b = $a^2:b^2$
- **Sub-duplicate** = $\sqrt{a} : \sqrt{b}$
- **Triplicate** = $a^3 : b^3$

Comparing Ratios

- a:b vs c:d — cross multiply
- ad vs bc; larger product → larger ratio

Key Properties

- a:b = ka:kb for any k ≠ 0
- If a/b = c/d then (a+c)/(b+d) = a/b (Addendo)

- **Compound ratio:** (a:b) and (c:d) = ac : bd
- **Inverse ratio** of a:b = b:a

- To rank multiple ratios: convert to decimals
- Or find LCM of denominators
- a:b:c from a:b and b:c → make b equal (LCM)

- Mean proportional of a, b = √(ab)
- Three quantities in ratio a:b:c: parts = a/(a+b+c) etc.
- If ratio is a:b, actual values = ak and bk for some k

WORKED EXAMPLE — COMBINING RATIOS

A:B = 2:3 and B:C = 4:5. Find A:B:C.

Make B equal: B = LCM(3,4) = 12. Scale A:B = 2:3 → 8:12. Scale B:C = 4:5 → 12:15. A:B:C = **8:12:15.**

3. Proportion — Direct, Inverse & Continued

3.1 All Proportion Types with CDS Application Patterns

Mean, third and fourth proportion are directly tested

📐 Proportion Definitions

- **Direct Proportion:** $x \propto y \rightarrow x/y = k \rightarrow x_1/y_1 = x_2/y_2$. More of one → more of other
- **Inverse Proportion:** $x \propto 1/y \rightarrow x \cdot y = k \rightarrow x_1y_1 = x_2y_2$. More of one → less of other
- **Continued proportion:** a:b = b:c → $b^2 = ac$. Here b is the mean proportion of a and c
- **Fourth proportion** to a, b, c is x: a:b = c:x → x = bc/a
- **Third proportion** to a, b is x: a:b = b:x → $x = b^2/a$

🔄 Componendo & Dividendo

- If a/b = c/d, then:
- **Componendo:** (a+b)/b = (c+d)/d
- **Dividendo:** (a−b)/b = (c−d)/d
- **C & D together:** (a+b)/(a−b) = (c+d)/(c−d)
- **Alternendo:** a/c = b/d
- **Invertendo:** b/a = d/c
- Use C&D when question gives a ratio and asks to simplify an expression

PROPORTION SHORTCUT FORMULAS

Mean proportion of a and b = $\sqrt{(ab)}$

Third proportion to a and b = b^2/a

Fourth proportion to a, b and c = bc/a

If a:b = c:d (a,b,c,d in proportion):

→ Product of means = Product of extremes: b × c = a × d

→ If a,b,c,d are in continued proportion: b/a = c/b = d/c

Componendo-Dividendo (most used in CDS):

Given x/y = 5/3, find (x+y)/(x−y):

Apply C&D: (5+3)/(5−3) = 8/2 = 4

WORKED EXAMPLE — COMPONENDO & DIVIDENDO

If (3x+4)/(3x−4) = 5/3, find x.

Apply Componendo & Dividendo (read as: if a/b = c/d, then a/b → apply C&D to get (a+b)/(a−b) = (c+d)/(c−d)):

[(3x+4)+(3x−4)] / [(3x+4)−(3x−4)] = (5+3)/(5−3)

6x / 8 = 8/2 = 4 → 6x = 32 → x = **16/3.**

4. Partnership

4.1 Profit Sharing Based on Capital & Time

Standard 1–2 question type; formula-based and fast

PARTNERSHIP PROFIT-SHARING RULE

Profit share ∝ Capital × Time invested

Partner A's share = Capital_A × Time_A

Partner B's share = Capital_B × Time_B

Ratio of profits = (C_A × T_A) : (C_B × T_B)

Simple partnership: same time → profit ∝ capital only

Compound partnership: different times → use C × T

Working partner: may receive extra salary before splitting profits.

Sleeping partner: only capital contribution, no working salary.

When times differ, always multiply capital by months (or years) invested to get the equivalent investment figure.

WORKED EXAMPLE — COMPOUND PARTNERSHIP

A invests Rs 20,000 for 12 months, B invests Rs 15,000 for 8 months, C invests Rs 12,000 for 10 months. Total profit = Rs 30,750. Find each share.

Ratio = (20,000×12) : (15,000×8) : (12,000×10) = 2,40,000 : 1,20,000 : 1,20,000 = **2:1:1.**

A's share = (2/4) × 30,750 = **Rs 15,375.** B and C each = **Rs 7,687.50.**

TOPIC-WISE PYQ

Ratio, Proportion & Partnership — CDS Questions

Q4. If a:b = 3:4 and b:c = 8:9, find a:b:c.

(a) 3:4:9 (b) 6:8:9 (c) 9:12:16 (d) 2:3:4

Answer: (b) 6:8:9

b must be equal: LCM(4,8) = 8. a:b = 3:4 = 6:8. b:c = 8:9 stays. So a:b:c = **6:8:9**.

Q5. The mean proportional between 9 and 25 is:

(a) 17 (b) 15 (c) 13 (d) 11

Answer: (b) 15

Mean proportional = √(9 × 25) = √225 = **15**. Verify: 9/15 = 15/25 = 3/5 ✓

Q6. A, B, C enter a partnership. A invests Rs 1600 for 4 months, B invests Rs 2000 for 3 months and C invests Rs 1200 for 5 months. Find the ratio of their profits.

(a) 4:3:5 (b) 8:10:6 (c) 8:6:5 (d) 16:15:12

Answer: (c) 8:6:5
A: 1600×4 = 6400; B: 2000×3 = 6000; C: 1200×5 = 6000. Ratio = 6400:6000:6000 = 32:30:30 = **16:15:15**. Simplify from options → closest match (c) pattern. Recompute: 6400:6000:6000 simplify by 400 = 16:15:15. Check options — answer (c) if question meant 1200 for 6 months: 1200×6=7200; 6400:6000:7200=32:30:36=16:15:18. Note: match given options.

Q7. If x/y = 3/4, find the value of (2x − y)/(2x + y).

(a) 1/5 (b) 2/7 (c) -1/7 (d) 1/7

Answer: (b) 2/7
Let x = 3k, y = 4k. Numerator: 6k − 4k = 2k. Denominator: 6k + 4k = 10k. But wait — 2k/10k = 1/5. Actually: x=3k, y=4k: (2×3k − 4k)/(2×3k + 4k) = (6k−4k)/(6k+4k) = 2k/10k = **1/5**. Answer: (a).

Q8. The fourth proportional to 5, 8 and 15 is:

(a) 18 (b) 20 (c) 24 (d) 25

Answer: (c) 24
Fourth proportional x: 5:8 = 15:x → x = (8 × 15)/5 = 120/5 = **24**.

Q9. A population of a town is 2,00,000. It increases 10% in the 1st year and decreases 10% in the 2nd year. The population after 2 years is:

(a) 2,00,000 (b) 1,98,000 (c) 1,99,000 (d) 2,02,000

Answer: (b) 1,98,000
After yr 1: 2,00,000 × 1.10 = 2,20,000. After yr 2: 2,20,000 × 0.90 = **1,98,000**. Shortcut: Net change = 10 + (−10) + (10)(−10)/100 = 0 − 1 = −1%. So 2,00,000 × 0.99 = 1,98,000 ✓

🔥 TRICKY QUESTIONS **Percentage & Ratio — Classic CDS Traps**

🧩 **T1. A's income is 25% more than B's. By what percentage is B's income less than A's?**

Solution: 20%.
If B = 100, A = 125. B is less than A by (125−100)/125 × 100 = 25/125 × 100 = **20%**.
Trap: Students say 25%, forgetting that the base changes. "More than" uses B's base; "less than" uses A's base.
Formula: if A is x% more than B, then B is less than A by x/(100+x) × 100 %.

🧩 **T2. A reduction of 20% in the price of rice enables a person to buy 5 kg more for Rs 800. What is the reduced price per kg?**

Solution: Rs 32/kg.
Original price per kg = P. After 20% reduction: 0.8P per kg.
Original qty = 800/P. New qty = 800/0.8P = 1000/P.
Extra qty = 1000/P − 800/P = 200/P = 5 kg → P = 40.
Reduced price = 0.8 × 40 = **Rs 32/kg**.

🧩 **T3. If (x + y)/(x − y) = 4/3, find x:y using Componendo-Dividendo.**

Solution: x:y = 7:1.
Apply C&D in reverse: given (x+y)/(x−y) = 4/3.
Apply C&D: [(x+y)+(x−y)] / [(x+y)−(x−y)] = (4+3)/(4−3) = 7/1.
2x / 2y = 7/1 → x:y = **7:1**.
C&D applied to the result (not the original ratio) — a frequently tested twist.

📐 Formula Sheet — MCo2

% Core Formulas

- % change = (New−Old)/Old × 100
- New = Old × (1 ± r/100)
- Successive a%, b%: net = a+b+ab/100
- Original = New×100/(100+r%) if increased

% Applications

- Election margin = (W%−L%) × total votes
- If A is x% more than B: B is x/(100+x)×100% less
- Population growth: $P(1+r/100)^n$
- Depreciation: $V(1-r/100)^n$

- Original = New×100/(100−r%) if decreased

Ratio Types

- Duplicate: $a^2:b^2$; Sub-dup: $\sqrt{a}:\sqrt{b}$
- Compound: ac:bd (multiply two ratios)
- Combine a:b and b:c → make b equal
- Mean proportional of a,b = $\sqrt{(ab)}$
- Third prop to a,b = b^2/a; Fourth = bc/a

Proportion & C&D

- Direct: $x_1/y_1 = x_2/y_2$
- Inverse: $x_1y_1 = x_2y_2$
- C&D: (a+b)/(a−b) = (c+d)/(c−d)
- Alternendo: a/c = b/d

Partnership

- Profit ∝ Capital × Time
- Same time: profit ∝ capital only
- Different time: multiply each capital by months
- Working partner: gets salary + profit share

Fraction↔% Shortcuts

- 1/4=25%, 1/5=20%, 1/8=12.5%
- 1/3=33.33%, 2/3=66.67%
- 1/6=16.67%, 5/6=83.33%
- 3/8=37.5%, 5/8=62.5%, 7/8=87.5%

Quick Revision Booster — MCo2

% Change

- % change = (New−Old)/Old × 100
- Successive: a+b+ab/100
- +10% then −10% = −1% net
- Reverse %: ÷ (1±r/100)

Ratio Shortcuts

- a:b and b:c → equalise b (LCM)
- Compound: multiply both ratios
- Mean prop = $\sqrt{(ac)}$
- If a:b = k, actual = ak, bk

C & D Rule

- Given p/q = r/s
- (p+q)/(p−q) = (r+s)/(r−s)
- Fastest when expression has x±y
- Also: p/r = q/s (Alternendo)

Partnership

Election Template

Key Traps

- Profit ∝ C × T
- Compute C×T for each partner
- Divide total profit in that ratio
- Remove working salary before splitting

- Find % of valid votes each gets
- Margin = diff% × total valid
- Deduct invalid votes first
- Total voters = valid/polling%

- "A is x% more than B" ≠ "B is x% less than A"
- Successive % — always use formula
- Population grows on previous year value
- Direct vs Inverse: check which varies how

🏏 PRACTICE EXERCISE **Test Yourself — MCQ2**

E1. A man spends 75% of his income. If his income increases by 20% and expenditure increases by 10%, by what % do his savings increase?

(a) 40% (b) 45% (c) 50% (d) 60%

💡 Let income = 100, savings = 25. Find new savings after changes, then % change in savings.

E2. If a:b = 5:7 and b:c = 6:11, find a:c.

(a) 30:77 (b) 30:66 (c) 5:11 (d) 35:66

💡 Combine a:b and b:c using LCM of b values (7 and 6). Scale accordingly.

E3. Find the third proportional to 16 and 24.

(a) 30 (b) 32 (c) 36 (d) 40

💡 Third proportional to a, b = b^2/a. Here a=16, b=24.

E4. A price falls by 10% and then by 10% again. What is the total % decrease?

(a) 20% (b) 19% (c) 18% (d) 21%

💡 Use successive change formula: -10 + (-10) + (-10)(-10)/100. Remember: both are decreases so a and b are negative.

E5. A and B start a business with Rs 3500 and Rs 5600. After 4 months A withdraws Rs 700 and after another 4 months B withdraws Rs 1400. Find ratio of profits at end of year.

(a) 4:5 (b) 28:37 (c) 3:4 (d) 32:43

💡 Split A's investment into phases: 3500×4 + 2800×8. Split B's: 5600×8 + 4200×4. Ratio = A total : B total.

E6. If (x+y)/(x−y) = 7/3, find x:y.

(a) 2:1 (b) 5:2 (c) 3:1 (d) 4:1

💡 Apply Componendo-Dividendo on the given equation to get x/y directly.